Real and Complex Continuous Fourier Transform

1 SINE/COSINE AND COMPLEX FOURIER SERIES

Convert the following sine/cosine Fourier series to $\mathbb{C}$-exponential form.

A) $5\sin(7x)$

B) $5\cos(2x)$

C) $7 + 2\sin(3x)$

D) $3\sin(4x) + 2\cos(4x)$

E) $4\sin(2x) + 5\cos(3x)$

F) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(2nx)$

Convert the following $\mathbb{C}$-exponential Fourier series to sine/cosine form.

G) $2e^{-2ti} + 2e^{2ti}$

H) $(-3i)e^{-2ti} + (3i)e^{2ti}$

I) $(2-3i)e^{-5ti} + (2+3i)e^{5ti}$

J) $(5+i)e^{-ti} + 1 + (5-i)e^{ti}$

K) $5e^{-3ti} + (7i)e^{-ti} + (-7i)e^{ti} + 5e^{3ti}$

L) $\displaystyle\sum_{-\infty}^{\infty} (-1)^n n\, i\, e^{2nti}$

2 FOURIER COEFFICIENTS AND SERIES

For the following functions (defined on $(-\pi,\pi)$ and extended periodically elsewhere)
(i) Find formulas for the $\mathbb{C}$-exponential Fourier coefficients.
(ii) Write the terms for $n = -1, 0, 1, 2$ in the $\mathbb{C}$-exponential Fourier series.
(Compute coefficients using the methods suggested – be careful about c_0.)

A) $f(t) = \delta(t)$ ("Impulse at $t = 0$")
Compute by using the definition.

B) $f(t) = \delta(t-c)$ ("Impulse at $t = c$")
Compute by shifting the answer from the previous part.

C) $f(t) = \delta(t-a) - \delta(t-b)$
Compute using linearity of Fourier series.

D) $f(t) = \begin{cases} 1 & a < t < b \\ 0 & \text{otherwise} \end{cases}$ ("Square Pulse Wave")
Compute by using the integral transformation on the previous answer.

E) $f(t) = \begin{cases} 1 & 0 < t < \pi \\ -1 & -\pi < t < 0 \end{cases}$ ("Up-Down Wave")
Compute using linearity of Fourier series.

F) $f(t) = |t|$ ("Zig-Zag Wave")
Compute by using the integral transformation on the previous answer.

G) $f(t) = 1$
Compute by using the definition.

H) $f(t) = t$ ("Saw-Tooth Wave")
Compute by using the definition.